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Fixed point theorems on a closed ball

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Abstract. The aim of the paper is to obtain some fixed point theorems for extended (φ, F)-weak type contraction on a closed ball in metric spaces. Our results generalize some recently established results.

1. Introduction

In 2012, Samet et al. [8] introduced a class of α-admissible mapping.

Definition 1 ([8]). Let $T : X \to X$ and $\alpha : X \times X \to [0, +\infty)$. We say that T is α-admissible if $x, y \in X$, $\alpha(x, y) \geq 1$ implies that $\alpha(Tx, Ty) \geq 1$.

Definition 2 ([7]). Let $T : X \to X$ and $\alpha, \eta : X \times X \to [0, +\infty)$ be two functions. We say that T is α-admissible mapping with respect to η if $x, y \in X$, $\alpha(x, y) \geq \eta(x, y)$ implies that $\alpha(Tx, Ty) \geq \eta(Tx, Ty)$.

If $\eta(x, y) = 1$, then Definition 2 reduces to Definition 1. If $\alpha(x, y) = 1$, then T is called an η-subadmissible mapping.

Definition 3 ([4]). Let (X, d) be a metric space. Let $T : X \to X$ and $\alpha, \eta : X \times X \to [0, +\infty)$ be two functions. We say that T is α-η-continuous mapping on (X, d) if for given $x \in X$ and sequence $\{x_n\}$ with

$$x_n \to x \text{ as } n \to \infty; \alpha(x_n, x_{n+1}) \geq \eta(x_n, x_{n+1}), \quad \text{for all } n \in \mathbb{N} \Rightarrow Tx_n \to Tx.$$

Definition 4 ([6]). Let (X, d) be a metric space. A mapping $T : X \to X$ is said to be an F-contraction if there exists $\tau > 0$ such that

$$(1) \qquad \forall\, x, y \in X,\ d(Tx, Ty) > 0 \Rightarrow \tau + F(d(Tx, Ty)) \leq F(d(x, y)),$$

where $F : \mathbb{R}_+ \to \mathbb{R}$ is a mapping satisfying the following conditions:

- (F_1) F is strictly increasing, i.e., for all $x, y \in \mathbb{R}_+$ such that $x < y$, $F(x) < F(y)$;
- (F_2) For each sequence $\{\alpha_n\}_{n=1}^{\infty}$ of positive numbers, $\lim_{n\to\infty} \alpha_n = 0$ if and only if $\lim_{n\to\infty} F(\alpha_n) = -\infty$;

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(F_3) There exists $k \in (0,1)$ such that $\lim_{\alpha\to 0^+} \alpha^k F(\alpha) = 0$.

We denote by Δ_F, the set of all functions satisfying the conditions (F_1)–(F_3).

Wardowski [9] modified Banach contraction principle for F-contraction as follows.

Theorem 1. *Let (X,d) be a complete metric space and let $T : X \to X$ be an F-contraction. Then T has a unique fixed point $z \in X$ and for every $x \in X$ the sequence $\{T^n x\}_{n\in\mathbb{N}}$ converges to z.*

Hussain et al. [4] introduced the following family of new functions. Let Δ_G denote the set of all functions $G : \mathbb{R}_+^4 \to \mathbb{R}_+$ satisfying:

(G) for all $t_1, t_2, t_3, t_4 \in \mathbb{R}_+$ with $t_1t_2t_3t_4 = 0$, there exists $\tau > 0$ such that $G(t_1, t_2, t_3, t_4) = \tau$.

Definition 5 ([4])**.** Let (X,d) be a metric space and T be a self-mapping on X. Also, suppose that $\alpha, \eta : X \times X \to [0,+\infty)$ are two function. We say that T is α-η-GF-contraction, if for $x, y \in X$ with $\eta(x,Tx) \le \alpha(x,y)$ and $d(Tx,Ty) > 0$, we have

$$G(d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)) + F(d(Tx,Ty)) \le F(d(x,y)),$$

where $G \in \Delta_G$ and $F \in \Delta_F$.

For $x \in X$ and $\epsilon > 0, \overline{B(x,\epsilon)} = \{y \in X : d(x,y) \le \epsilon\}$ is a closed ball in (X,d). The following result, regarding the existence of the fixed point of the mapping satisfying a contractive condition on the closed ball, was given in [5]. The result is very useful in the sense that it requires the contraction condition only on a closed ball, instead of on the whole space.

Theorem 2 ([5])**.** *Let (X,d) be a complete metric space, $T : X \to X$ be a mapping, $r > 0$ and x_0 be an arbitrary point in X. Suppose there exists $k \in [0,1)$ with*

$$d(Tx,Ty) \le k\, d(x,y), \quad \text{for all } x,y \in Y = \overline{B(x_0,r)},$$

and $d(x_0,Tx_0) < (1-k)r$. Then there exists a unique point $x*$ in $\overline{B(x_0,r)}$ such that $x = Tx*$.*

Recently, in 2019, Hussain [3] introduced the Ćirić type modified F-contraction on a closed ball in a complete metric space.

Definition 6 ([3])**.** Let (X,d) be a metric space. A self-mapping $T : X \to X$ is said to be a modified F-contraction via α-admissible mappings if there exists $\tau > 0$ such that

$$d(Tx,Ty) > 0 \Rightarrow \tau + F(\alpha(x,y)d(Tx,Ty)) \le F(\psi(M(x;y))),$$

where

$$M(x,y) = \max\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty) + d(y,Tx)}{2}\}$$

for all $x, y \in \overline{B(x_0, r)} \subseteq X$; where $F : \mathbb{R}^+ \to \mathbb{R}$ is a mapping satisfying $(F1) - (F3)$ and $\psi \in \Psi$.

In Definition 6, Ψ be the familiy of functions of self-mappings on $[0, \infty)$ satisfying:

(i) ψ is nondecreasing.
(ii) $\Sigma_{n=1}^{\infty} \psi^n(t) < +\infty$, for each $t > 0$.

Remark 1. If $\psi \in \Psi$, then $\psi(t) < t$ for all $t > 0$.

Using Definition 6, Hussain [3] obtained the following result.

Theorem 3 ([3]). *Let (X, d) be a complete metric space. Let $T : X \to X$ be a modified F-contraction via α-admissible mappings and x_0 be an arbitrary point in X. Assume that*

$$x, y \in \overline{B(x_0, r)}, \quad \tau + F(\alpha(x, y)d(Tx, Ty)) \leq F(\psi(M(x, y))), \tag{2}$$

where $\tau > 0$. Moreover

$$\Sigma_{j=0}^{N} d(x_0, Tx_0) \leq r, \text{ for all } j \in \mathbb{N} \text{ and } r > 0.$$

Suppose that the following assertions hold:

(i) *T is an α-admissible mapping;*
(ii) *there exist a point $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$;*
(iii) *T is continuous.*

Then there exist a point $x*$ in $\overline{B(x_0, r)}$ such that $Tx = x*$.*

In this paper, we obtain some fixed point results which generalize the results of Dey et al [1], Dung and Hang [2] and Hussain [3] on a closed ball in a complete metric space.

2. Main Results

Now, we introduce the following definition.

Definition 7. Let (X, d) be a metric space. A self-mapping $T : X \to X$ is said to be a modified F-contraction II via α-admissible mappings if there exists $\tau > 0$ such that

$$d(Tx, Ty) > 0 \Rightarrow \tau + F(\alpha(x, y)d(Tx, Ty)) \leq F(\psi(M(x, y))); \tag{3}$$

where,

$$M(x, y) = \max\Big\{d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{2}, \frac{d(T^2x, x) + d(T^2x, Ty)}{2}, d(T^2x, Tx), d(T^2x, y), d(T^2x, Ty)\Big\},$$

for all $x, y \in \overline{B(x_0, r)} \subseteq X$, and $F : \mathbb{R}_+ \to \mathbb{R}$ is a mapping satisfying (F_1)–(F_3) and $\psi \in \Psi$, where Ψ is defined as the same in Definition 6.

Theorem 4. *Let (X,d) be a complete metric space and $T : X \to X$ a modified F-contraction II via α-admissible mappings and x_0 be an arbitrary point in X. Assume that*

$$x, y \in \overline{B(x_0, r)}, \quad \tau + F(\alpha(x,y)d(Tx,Ty)) \le F(\psi(M(x,y))), \tag{4}$$

where $\tau > 0$. Moreover,

$$\Sigma_{j=0}^{N} d(x_j, Tx_j) \le r, \quad \forall j \in \mathbb{N} \text{ and } r > 0.$$

Suppose that the following assertions hold:

(i) *T is an α-admissible mapping;*
(ii) *there exist a point $x_0 \in X$ such that $\alpha(x_0, Tx_0) \ge 1$;*
(iii) *T is continuous.*

Then there exist a point $x*$ in $\overline{B(x_0, r)}$ such that $Tx = x*$.*

Proof. Due to assumption (ii), there exist a point $x_0 \in X$ such that $\alpha(x_0, Tx_0) \ge 1$. Now, we construct a sequence $\{x_n\}_{n\ge 0}$ in X such that $x_{n+1} = Tx_n$. $\{x_n\}$ is a non-increasing sequence. If we assume that $x_n = x_{n+1}$ for some $n \ge 0$, then the proof is complete obviously. So, we assume that $x_n \ne x_{n+1}$ for all $n \ge 0$. Since $\alpha(x_0, x_1) = \alpha(x_0, Tx_0) \ge 1$ and T is α-admissible, we have

$$\alpha(x_n, x_{n+1}) \ge 1, \quad \forall n \ge 0. \tag{5}$$

Firstly, we show that $x_n \in \overline{B(x_0, r)}$ for all $n \in \mathbb{N}$. For this, consider $d(x_0, x_1) = d(x_0, Tx_0) \le r$. Thus $x_1 \in \overline{B(x_0, r)}$. Suppose that $x_2, \ldots, x_j \in \overline{B(x_0, r)}$ for some $j \in \mathbb{N}$, then from (4),

$$\begin{aligned} F(\alpha(x_{j-1}, x_j)d(Tx_{j-1}, Tx_j)) &\le F(\psi(M(x_{j-1}, x_j)) - \tau \\ \Rightarrow d(x_j, x_{j+1}) &< \psi((M(x_{j-1}, x_j))) < M(x_{j-1}, x_j) \end{aligned}$$

where

$$\begin{aligned} M(x_{j-1}, x_j) &= \max\Big\{ d(x_{j-1}, x_j), d(x_{j-1}, x_j), d(x_j, x_{j+1}), \\ &\qquad \frac{d(x_{j-1}, x_{j+1}) + d(x_j, x_j)}{2}, \frac{d(x_{j+1}, x_{j-1}) + d(x_{j+1}, x_{j+1})}{2}, \\ &\qquad d(x_{j+1}, x_j), d(x_{j+1}, x_j), d(x_{j+1}, x_{j+1}) \Big\} \\ &= \max\big\{ d(x_{j-1}, x_j), d(x_j, x_{j+1} \big\}. \end{aligned}$$

Therefore, we have

$$\begin{aligned} F(d(x_j, x_{j+1}) &\le F(\alpha(x_{j-1}, x_j)d(Tx_{j-1}, Tx_j)) \\ &\le F(\max\{d(x_{j-1}, x_j), d(x_j, x_{j+1})\}) - \tau. \end{aligned}$$

If $\max\{d(x_{j-1}, x_j), d(x_j, x_{j+1})\} = d(x_j, x_{j+1})$, then

$$\Rightarrow F(d(x_j, x_{j+1}) \le F(d(x_j, x_{j+1})) - \tau.$$

This gives $\tau \leq 0$, a contradiction. Hence, $\max\{d(x_{j-1}, x_j), d(x_j, x_{j+1})\} = d(x_{j-1}, x_j)$. Now,

$$\begin{aligned} d(x_0, x_{j+1}) &\leq d(x_0, x_1) + \ldots + d(x_j, x_{j+1}) \\ &= \Sigma_{j=0}^{N} d(x_j, Tx_j) \leq r. \end{aligned}$$

Therefore, $x_{j+1} \in \overline{B(x_0, r)}$ for all $n \in \mathbb{N}$. Continuing this process, we get

$$\begin{aligned} F(d(x_n, x_{n+1})) &\leq F(d(x_{n-1}, x_n)) - \tau \\ &= F(d(Tx_{n-2}, Tx_{n-1})) - \tau \\ &\leq F(d(x_{n-2}, x_{n-1})) - 2\tau \\ &\vdots \\ &\leq F(d(x_0, x_1)) - n\tau. \end{aligned}$$

This implies

$$F(d(x_n, x_{n+1})) \leq F(d(x_0, x_1)) - n\tau. \tag{6}$$

Taking limit we get, $\lim_{n\to\infty} F(d(x_n, x_{n+1})) = -\infty$. So, we have

$$d(x_n, x_{n+1}) = 0. \tag{7}$$

From $(F3)$, there exists $k \in (0, 1)$ such that

$$\lim_{n\to\infty} (d(x_n, x_{n+1}))^k F(d(x_n, x_{n+1})) = 0. \tag{8}$$

From (6), for all $n \in N$, we obtain

$$\begin{aligned} (d(x_n, x_{n+1}))^k (F(d(x_n, x_{n+1})) - F(d(x_0, x_1))) \leq \\ -(d(x_n, x_{n+1}))^k n\tau \leq 0. \end{aligned} \tag{9}$$

By using (7), (8) and letting $n \to \infty$ in (9), we have

$$\lim_{n\to\infty} (n(d(x_n, x_{n+1}))^k) = 0. \tag{10}$$

We observe that from (10), there exist $n_1 \in \mathbb{N}$ such that $n(d(x_n, x_{n+1}))^k \leq 1$ for all $n \geq n_1$, we get

$$d(x_n, x_{n+1}) \leq \frac{1}{n^k}, \quad \forall n \geq n_1. \tag{11}$$

Now $m, n \in \mathbb{N}$ such that $m > n \geq n_1$. Then by triangle inequality and from (11), we have

$$\begin{aligned} d(x_n, x_m) &\leq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots + d(x_{m-1}, x_m) \\ &= \Sigma_{i=n}^{m-1} d(x_i, x_{i-1}) \\ &\leq \Sigma_{i=n}^{\infty} d(x_i, x_{i+1}) \\ &\leq \Sigma_{i=n}^{\infty} \frac{1}{i^{\frac{1}{k}}}. \end{aligned} \tag{12}$$

The series $\frac{1}{i^{\frac{1}{k}}}$ is convergent. Taking the limit as $n \to \infty$, in (12), we have $\lim_{n,m\to\infty} d(x_n, x_m) = 0$. Hence x_n is a Cauchy sequence. Since, X is a complete metric space there exists an $x* \in \overline{B(x_0, r)}$ such that $x_n \to x*$ as $n \to \infty$. T is a continuous then $x_{n+1} = Tx_n \to Tx*$ as $n \to \infty$. That is, $x* = Tx*$. Hence $x*$ is a fixed point of T. □

Motivating by the paper [1], we introduce the following definition.

Definition 8. Let (X, d) be a metric space. A self-mapping $T : X \to X$ is said to be a modified F-contraction III via α-admissible mappings if there exists $\tau > 0$ such that

$$(13) \quad d(Tx, Ty) > 0 \quad \Rightarrow \quad \tau + F(\alpha(x, y)d(Tx, Ty)) \leq F(\psi(M'(x, y))),$$

where

$$M'(x, y) = \max\Big\{d(x, y), \frac{d(x, Ty) + d(y, Tx)}{2}, \frac{d(T^2x, x) + d(T^2x, Ty)}{2}, d(T^2x, Tx), d(T^2x, y), d(Tx, y) + d(y, Ty), d(T^2x, Ty) + d(x, Tx)\Big\},$$

for all $x, y \in \overline{B(x_0, r)} \subseteq X$, and $F : \mathbb{R}_+ \to \mathbb{R}$ is a mapping satisfying (F_1)-(F_3) and $\psi \in \Psi$, where Ψ is defined as the same in Definition 6.

Remark 2. Every modified F-contraction III is a modified F-contraction II via α-admissible mapping. The reverse implications do not hold.

Now we obtain the following result which is a generalization of Theorem 4.

Theorem 5. *Let (X, d) be a complete metric space and $T : X \to X$ a modified F-contraction III via α-admissible mappings and x_0 be an arbitrary point in X. Assume that*

$$(14) \qquad x, y \in \overline{B(x_0, r)}, \quad \tau + F(\alpha(x, y)d(Tx, Ty)) \leq F(\psi(M'(x, y))),$$

where $\tau > 0$. Moreover,

$$\Sigma_{j=0}^{N} d(x_j, Tx_j) \leq r, \quad \forall j \in \mathbb{N} \text{ and } r > 0.$$

Suppose that the following assertions hold:

(i) *T is an α-admissible mapping;*
(ii) *there exist a point $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$;*
(iii) *T is continuous.*

Then there exist a point $x*$ in $\overline{B(x_0, r)}$ such that $Tx = x*$.*

Proof. The proof is same as in Theorem 4. □

Remark 3. Theorem 4 and 5 generalize the main result of Hussain [3] and also extends results of [2] and [1] on closed ball in a complete metric space.

3. Fixed Point Theorems for GF-Contraction on Closed Ball

Definition 9. Let T be a self mapping in a metric space (X, d) and let x_0 be an arbitrary point in X. Also suppose that $\alpha : X \times X \to -\infty \cup (0, +\infty); \eta : X \times X \to \mathbb{R}_+$ are two functions. We say that T is called modified $\alpha - \eta - \psi$-GF-contraction II on closed ball if for all $x, y \in \overline{B(x_0, r)} \subseteq X$; with $\eta(x, Tx) \leq \alpha(x, y)$ and $d(Tx, Ty) > 0$; we have

$$G\big(d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\big) + F\big(d(Tx,Ty)\big) \leq \\ \leq F\big(\psi(M(x,y))\big), \tag{15}$$

where

$$M(x,y) = \max\Big\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty)+d(y,Tx)}{2}, \\ \frac{d(T^2x,x)+d(T^2x,Ty)}{2}, d(T^2x,Tx), d(T^2x,y), d(T^2x,Ty)\Big\}.$$

Moreover,

$$\Sigma_{j=0}^{N} d(x_j, Tx_j) \leq r, \quad \forall j \in \mathbb{N} \text{ and } r > 0,$$

$G \in \Delta_G, \psi \in \Psi$, and $F \in \Delta_F$.

Theorem 6. *Let (X, d) be a complete metric space. Let $T : X \to X$ be an α-η-ψ-GF-contraction II mapping on closed ball satisfying the following assertions:*

(i) *T is an α-admissible mapping with respect to η;*
(ii) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq \eta(x_0, Tx_0)$;*
(iii) *T is α-η-continuous.*

Then there exist a point $x*$ in $\overline{B(x_0, r)}$ such that $Tx = x*$.*

Proof. Let $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq \eta(x_0, Tx_0)$. For $x_0 \in X$, we construct a sequence $\{x_n\}_{n=1}^{\infty}$ such that $x_1 = Tx_0, x_2 = Tx_1 = T^2x_0$. Continuing this way, we have $x_{n+1} = Tx_n = T^{n+1}x_0$, $\forall n \in \mathbb{N}$.

Since T is an α-admissible mapping with respect to η, then $\alpha(x_0, x_1) = \alpha(x_0, Tx_0) \geq \eta(x_0, Tx_0)$. Continuing this process, we have

$$\eta(x_{n-1}, Tx_{n-1}) = \eta(x_{n-1}, x_n) \leq \alpha(x_{n-1}, x_n), \quad \forall n \in \mathbb{N}.$$

If there exists an $n \in \mathbb{N}$ such that $d(x_n, Tx_n) = 0$. We assume that $x_n \neq x_{n+1}$ with

$$d(Tx_{n-1}, Tx_n) = d(x_n, Tx_n) > 0, \quad \forall n \in \mathbb{N}.$$

First we show that $x_n \in \overline{B(x_0, r)}, \quad \forall n \in \mathbb{N}$,

$$d(x_0, x_1) = d(x_0, Tx_0) \leq r.$$

Thus, $x_1 \in \overline{B(x_0, r)}$. Suppose that $x_2, \ldots, x_j \in \overline{B(x_0, r)}$ for some $j \in \mathbb{N}$. Since, T is an $\alpha - \eta - \psi$-GF-contraction on closed ball, such that

$$G\big(d(x_{j-1},Tx_{j-1}),d(x_j,Tx_j),d(x_{j-1},Tx_j),d(x_j,Tx_{j-1})\big) \\ +F\big(d(Tx_{j-1},Tx_j)\big)\leq F\big(\psi(M(x_{j-1},x_j))\big). \tag{16}$$

This implies

$$G\big(d(x_{j-1},x_j),d(x_j,x_{j+1}),d(x_{j-1},x_{j+1}),0\big) \\ +F\big(d(x_j,x_{j+1})\big)\leq F\big(\psi(M(x_{j-1},x_j))\big). \tag{17}$$

Since, $d(x_{j-1},x_j)\cdot d(x_j,x_{j+1})\cdot d(x_{j-1},x_{j+1})\cdot 0=0$, then there exist a $\tau>0$ such that

$$F\big(d(x_j,x_{j+1})\big)=F\big(d(Tx_{j-1},Tx_j)\big)\leq F\big(\psi(M(x_{j-1},x_j))\big)-\tau.$$

The rest of the proof follows from the proof of the Theorem 4. □

Along the same lines we introduce the modified $\alpha-\eta-\psi$-GF-contraction III on a closed ball.

Definition 10. Let T be a self mapping in a metric space (X,d) and let x_0 be an arbitrary point in X. Also suppose that $\alpha: X\times X\to -\infty\cup(0,+\infty);\eta: X\times X\to\mathbb{R}_+$ are two functions. We say that T is called modified $\alpha-\eta-\psi$-GF-contraction III on a closed ball if for all $x,y\in\overline{B(x_0,r)}\subseteq X$; with $\eta(x,Tx)\leq\alpha(x,y)$ and $d(Tx,Ty)>0$; we have

$$G\big(d(x,Tx),d(y,Ty),d(x,Ty),d(y,Tx)\big) \\ +F\big(d(Tx,Ty)\big)\leq F\big(\psi(M'(x,y))\big), \tag{18}$$

where

$$M'(x,y)=\max\Big\{d(x,y),\frac{d(x,Ty)+d(y,Tx)}{2},\frac{d(T^2x,x)+d(T^2x,Ty)}{2}, \\ d(T^2x,Tx),d(T^2x,y),d(Tx,y)+d(y,Ty),d(T^2x,Ty)+d(x,Tx)\Big\},$$

Moreover,

$$\Sigma_{j=0}^{N}d(x_j,Tx_j)\leq r,\quad\forall j\in\mathbb{N}\text{ and }r>0,$$

$G\in\Delta_G,\psi\in\Psi$, and $F\in\Delta_F$.

Now, we obtain the following generalization of Theorem 6.

Theorem 7. *Let (X,d) be a complete metric space. Let $T: X\to X$ be an α-η-ψ-GF-contraction III mapping on closed ball satisfying the following assertions:*

(i) *T is an α-admissible mapping with respect to η;*
(ii) *there exists $x_0\in X$ such that $\alpha(x_0,Tx_0)\geq\eta(x_0,Tx_0)$;*
(iii) *T is α-η-continuous.*

Then there exist a point $x*$ in $\overline{B(x_0,r)}$ such that $Tx*=x*$.

Proof. The proof is same as in Theorem 6. □

Remark 4. Theorem 6 and 7 generalize Theorem 3.2 of [3].

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